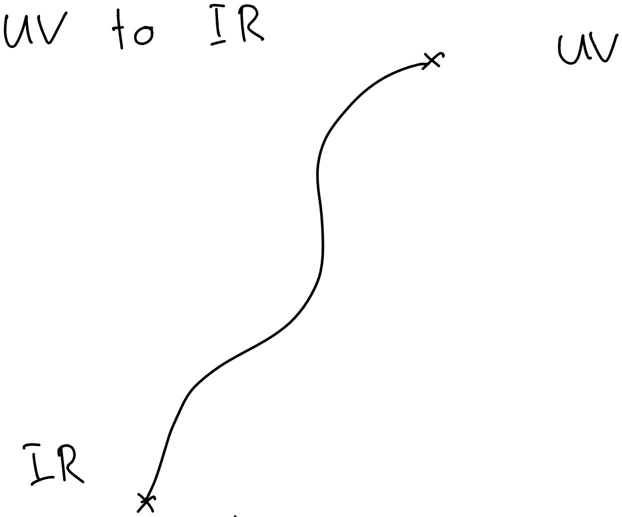
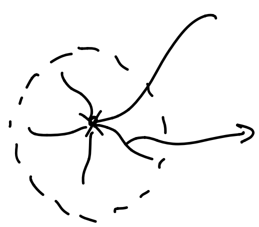
§2. Deformations of Superconformal Theories

QFT's are renormalization group (RG) flows from UV to IR



fixed-point $\longrightarrow$ CFT

Given a fixed point, we want to analyze its deformations



$\longrightarrow$ flows initiated by deformations

3 classes:

1) Adding local operators to the Lagrangian

$$\delta \mathcal{L} = g\, \mathcal{O}$$

running coupling constant

local operator defined at $g=0$

2) Gauging a global symmetry:

gauge a continuous flavor symmetry with conserved current j_μ

$\rightarrow$ may be obstructed by anomalies

3) Moving onto a moduli space of vacua:

In $d > 2$ spacetime dimensions CFT may possess non-trivial moduli space of vacua.

$\rightarrow$ breaks conformal symmetry spontaneously (restored at origin)

We will focus on deformation 1)

$\mathcal{O}$ must reside in representations of $SO(d,2)$

$\rightarrow$ labelled by weights under $SO(d) \times SO(2)$

Have unique operator $\mathcal{O}$ of lowest dimension Δ_0

Known as conformal primary

$$K_\mu \mathcal{O} = 0 \qquad\qquad P_\mu \mathcal{O}$$

K_μ: of scaling dimension -1

$P_\mu \mathcal{O}$: descendants of scaling dimension $\Delta_0 + 1$ $(P_\mu \sim \partial_\mu)$

$\mathcal{O}$ cannot be written as $\partial_\mu \mathcal{O}'$

Let us denote Δ_0 and $SO(d)$ weights of $\mathcal{O}$ by $L_\mathcal{O}$. There is natural inner product $\langle \mathcal{O}' | \mathcal{O}'' \rangle$

require $\langle \mathcal{O}' | \mathcal{O}'' \rangle \geq 0$

$\rightarrow$ unitarity bound $\Delta_0 \geq f(L_{\mathcal{O}})$

$\rightarrow$ null-states when bound is saturated

A deformation $\delta \mathcal{L} \sim g \mathcal{O}$ must be

- a conformal primary
  (descendants $\partial_\mu \mathcal{O}$ lead to total derivatives)
- $SO(d)$ scalar $\rightarrow$ preserves Lorentz symmetry

$\rightarrow \quad \langle \Box \mathcal{O} | \Box \mathcal{O} \rangle \geq 0 \Rightarrow \Delta_{\mathcal{O}} \geq \frac{d-2}{2}$

(recall $\Delta_{\mathcal{O}} = \underset{\substack{\uparrow \\ \text{highest weight of } SO(d)}}{h_1} + \frac{d-2}{2}$)

$S(d-2)$-weights free

bound is saturated for $\Box \mathcal{O} = 0$

Deformation types:

- "relevant deformations" ($\Delta_{\mathcal{O}} < d$):
  CFT at $g=0$ UV fixed point of RG-flow
  $\rightarrow$ g grows in the IR (perturbation theory breaks down)
- "Irrelevant deformations" ($\Delta_{\mathcal{O}} > d$):
  CFT is at IR fixed-point of RG-flow
  $\rightarrow$ g flows to zero
- "Marginal deformations" ($\Delta_{\mathcal{O}} = d$):
  preserve conformal invariance $\rightarrow$ lead to nearby fixed point

superconformal theories:

Want to look at deformations preserving Poincaré Q-supersymmetries (not necessarily S-symmetries)

Recall that we have the following SCA's:

$d=3$ $\quad osp(\mathcal{N}|4) \supset SO(3,2) \times SO(\mathcal{N})_R$

$d=4$ $\quad \begin{cases} sl(2,2|\mathcal{N}) \supset SO(4,2) \times SU(\mathcal{N})_R \times U(1)_R, & \mathcal{N} \neq 4 \\ psl(2,2|4) \supset SO(4,2) \times SU(4)_R, & \mathcal{N}=4 \end{cases}$

$d=5$ $\quad F(4) \supset SO(5,2) \times SU(2)_R, \quad \mathcal{N}=1$

$d=6$ $\quad osp(6,2|\mathcal{N}) \supset so(6,2) \times sp(2\mathcal{N})_R$

$\mathcal{N}$ denotes number of d-supercharges

N_Q denotes total number of supercharges

In $d=3,4,5,6$ minimal $\mathcal{N}=1$ supersymmetry corresponds to $N_Q = 2,4,8,8$

Notation: Will consider consider conformal primaries labelled by $L_\mathcal{O}$, $R_\mathcal{O}$, $\Delta_\mathcal{O}$

Lorentz-charge $\quad$ R-sym charge $\quad$ scaling dim

$$\mathcal{O} \in [L_\mathcal{O}]^{(R_\mathcal{O})}_{\Delta_\mathcal{O}}$$

superconformal primaries:

a conformal primary $\mathcal{V}$ with

- lowest scaling dimension $\Delta_{\mathcal{V}}$
- irreducible under R-sym

$\rightarrow$ annihilated by S (scaling dim $-\frac{1}{2}$) and K_μ (scaling dim. -1)

other conformal primaries are superconformal descendants of $\mathcal{V}$ (by action of Q)

Unitarity bound:

$$\Delta_{\mathcal{V}} \geq f(L_{\mathcal{V}}, R_{\mathcal{V}})$$

saturation leads to null-states

$\rightarrow$ descendants form sub-representation

notation: "short" representation

Deformations:

conformal primary $\mathcal{O}$ with

$$Q\,\mathcal{O} = \partial_\mu(\cdots)$$

$\rightarrow$ must transform to descendant under action of Q

$\rightarrow$ cannot be superconformal primary

$\rightarrow$ must be a "top" component

$Q^\ell \mathcal{V}$ are "level" ℓ conformal primaries (ℓ nested anti-commutators)

$$\{Q_i, Q_j\} \sim 0, \quad i,j=1,\dots,N_Q$$

(we are dropping ∂_μ-terms, correspond to conformal descendants)

We have:

$0 \leq l \leq l_{max}$ with $l_{max} \leq N_Q$

→ saturated for "long" multiplets (no "null"-states)

$Q^{N_Q} \mathcal{V}$ is top component

Lorentz- and R-sym singlet

"D-term" deformation $\mathcal{L}_D = Q^{N_Q} \mathcal{V}$

$\Delta(\mathcal{L}_D) = \frac{1}{2} N_Q + \Delta_{\mathcal{V}}$

example: Kähler potential $\int d^4\theta \, K(\Phi, \bar{\Phi})$ in $d=4$ $(\mathcal{N}=1)$

→ F-terms correspond to "short" multiplets (BPS)

$\mathcal{V}_{BPS}$ is annihilated by half of supercharges

$\mathcal{L}_F = Q^{\frac{1}{2}N_Q} \mathcal{V}_{BPS}$

example: chiral superpotential $\int d^2\theta \, W$

→ sporadic very short multiplets

example: consider the stress-tensor multiplet in

$\mathcal{N}=4$ $d=3$ SCFT's

Lorentz-sym.: $SU(2)$

R-sym.: $SU(2)_R \times SU(2)'_R$

→ $[j]_\Delta^{(R;R')}$

$Q^{i,i'}_{\alpha}$ transform as $[1]^{(1;1)}_{1/2}$ $\rightarrow$ conformal prim. decomp.:

$$\boxed{[0]^{(0;0)}_{1}} \xrightarrow{Q} \boxed{[1]^{(1;1)}_{3/2}} \xrightarrow{Q} \boxed{\underbrace{[2]^{(2;0)}_{2} \oplus [2]^{(0;2)}_{2}}_{\text{R-sym. currents}} \oplus [0]^{(0;0)}_{2}}$$

$$\rightarrow \underset{\text{supersymmetry current}}{\boxed{[3]^{(1;1)}_{5/2}}} \xrightarrow{Q} \underset{\text{stress-tensor}}{\boxed{[4]^{(0;0)}_{3}}}$$

$[0]^{(0;0)}_{2}$ at level 2 is Lorentz-scalar

$$Q\,[0]^{(0;0)}_{2} \sim 0$$

$\rightarrow$ $[0]^{(0;0)}_{2}$ gives relevant deformation with scaling dimension $\Delta = 2$.

Table of deformations in various dimensions:

see next page

d	N	Relevant	Marginal	Irrelevant Δ_{min}
3	1	D-term	D-term	$\Delta_{min} > 3$
	2	Flavor current, F-t.	F-term	$\Delta_{min} > 3$
	3	Flavor current	—	4
	4	stress t., fl. curr.	—	4
	5, 6	stress tensor	—	5
	8	stress tensor	—	6
4	1	F-term	F-term	$\Delta_{min} > 4$
	2	fl. curr., F-term	F-term	$\Delta_{min} > 4$
	3	—	—	$\Delta_{min} > 4$
	4	—	stress tensor	8
5	1	flavor curr.	—	8
6	(1,0)	—	—	10
	(2,0)	—	—	12